

Holomorphic Supersymmetric Nambu–Jona-Lasinio Model with Application to Dynamical Electroweak Symmetry Breaking

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Based on our idea of an alternative supersymmetrization of the Nambu–Jona-Lasinio model for dynamical symmetry breaking, we analyze the resulting new model with a holomorphic dimension-five operator in the superpotential. The approach provides a new direction for modeling dynamical symmetry breaking in a supersymmetric setting. In particular, we adopt the idea to formulate a model that gives rise to the Minimal Supersymmetric Standard Model as the low energy effective theory with both Higgs superfields as composites. A renormalization group analysis is performed to establish the phenomenological viability of the scenario, with admissible background scale that could go down to the TeV scale. We give the Higgs mass range predicted.

I. INTRODUCTION

The Nambu–Jona-Lasinio (NJL) model [1] is a classic model on dynamical symmetry breaking. A dimension-six operator of four-fermion interaction is used to induce a bi-fermion vacuum condensate. The bi-fermion configuration behaves as a scalar composite; that is, the effective Higgs field responsible for symmetry breaking and Dirac fermion mass. The possibility of a different mechanism for the electroweak symmetry breaking is a very interesting and inspiring option.

For the NJL model to give the vacuum condensate, the four-fermion interaction needs to have a strong enough coupling. With the effective Higgs multiplet introduced as an auxiliary field, the strong four-fermion coupling translates into a large Yukawa coupling realizing plausibly the heavy top quark phenomenologically. Such a top condensate model was constructed in 1989 [2–4]. We refer readers to Ref. [5] for a comprehensive review of the details of the model and other related topics.

The investigation of supersymmetric version of the NJL model was started in the 1980s. As a direct supersymmetrization, a dimension-six operator that has a four-fermion interaction in its D term was introduced [6]. It was realized that nontrivial vacuum is not possible due to supersymmetric cancelation, unless soft supersymmetry (SUSY) breaking is incorporated. The scheme introduces two new chiral superfields in the low energy effective field theory with asymmetric roles. The approach leads to the most popular candidate theory beyond the Standard Model (SM) — the Minimal Supersymmetric Standard Model (MSSM) as the low energy effective field theory, with interesting relations among some of the model parameters [7, 8]. Compared to the non-SUSY model, it improves on or eliminates a fine-tuning problem on the four-fermion coupling while allowing the lower top mass. Incorporating the NJL mechanism into the MSSM has the advantage of leaving the model superfield spectrum with only the part that is strongly constrained by the gauge symmetries, without the otherwise unconstrained vectorlike pair of Higgs superfields, besides en-

riching the naive Higgs mechanism with a conceptually more appealing dynamical structure. Phenomenological study of the model scenario was implemented with the infrared quasi-fixed-point (IQFP) solution for the top quark mass [8]. In a similar spirit, Ref. [9] presents an IQFP determination of all third generation fermion masses in the MSSM. That IQFP scenario should correspond to having both Higgs superfields as composites, which is not compatible with the conventional supersymmetric Nambu–Jona-Lasinio (SNJL) model.

Here, we propose a holomorphic variant of the SNJL model. Instead of a dimension-six operator, we consider a holomorphic dimension-five one in the superpotential. We illustrate how the scenario may be used for dynamical symmetry breaking. While the dimension-five term does not contain the four-fermion interaction as a component, the model does have other features that resemble the non-SUSY NJL model more closely compared to the old SNJL model. We discuss how a model of such kind can give rise to the MSSM as the low energy effective field theory realizing the IQFP solution of Ref. [9]. To fully accommodate the masses and mixing of the quarks in the MSSM, we find the kind of dimension-five four-superfield interactions actually cannot be avoided even in the old SNJL model. However, none of the dimension-five terms plays a role in inducing symmetry breaking in that case.

Inspired by the holomorphic model, we implemented a renormalization group (RG) analysis of the MSSM numerically, looking for compatibility with the model scenario. One would think that the current top mass of 171.3 ± 1.6 GeV is very difficult to be accommodated by a NJL model. It is not the case for our holomorphic SNJL model scenario. With a large enough value for $\tan\beta$, fitting the experimental quark masses presents little problem. And somewhat to our surprise, we realize that the bottom quark Yukawa coupling plays a more important role than the top Yukawa, and the composite scale could be very low. With a simple calculation, the Higgs mass turns up close to the current bounds. Note that, phenomenologically, fitting the current top mass with the MSSM from the old SNJL model actually

pushes the $\tan\beta$ value into a narrow window between 0.5 to 1.5 which is essentially excluded by the LEP result. Our holomorphic version as the first complete model for the MSSM, however, is phenomenologically viable.

We conclude that the holomorphic SNJL model idea provides an interesting alternative to the construction of dynamical symmetry breaking models and the case for such a model as what is behind the MSSM and the hence the phenomenology at the LHC scale is worth a more serious investigation.

II. THE HOLOMORPHIC SNJL MODEL

Consider the following model Lagrangian :

$$\mathcal{L} = \int d^4\theta [\bar{\Phi}_+\Phi_+ + \bar{\Phi}_-\Phi_-] - \int d^2\theta \frac{G}{2} \Phi_+\Phi_-\Phi_+\Phi_- + h.c. . \quad (1)$$

Besides the kinetic terms, we have introduced a four-superfield interaction term, but it is in the superpotential hence of dimension-five only. In the simplest version of the model, we need only one auxiliary chiral superfield Φ_0 to rewrite the Lagrangian (1) with a Yukawa coupling. Explicitly, we consider

$$\begin{aligned} \mathcal{L} &= \int d^4\theta [(\bar{\Phi}_+\Phi_+ + \bar{\Phi}_-\Phi_-)(1 - m^2\theta^2\bar{\theta}^2)] \\ &\quad + \left\{ \int d^2\theta \left[\frac{1}{2}(\sqrt{\mu}\Phi_0 + \sqrt{G}\Phi_+\Phi_-)(\sqrt{\mu}\Phi_0 + \sqrt{G}\Phi_+\Phi_-) \right. \right. \\ &\quad \left. \left. - \frac{G}{2}\Phi_+\Phi_-\Phi_+\Phi_- \right] + h.c. \right\} \\ &= \int d^4\theta [(\bar{\Phi}_+\Phi_+ + \bar{\Phi}_-\Phi_-)(1 - m^2\theta^2\bar{\theta}^2)] \\ &\quad + \left\{ \int d^2\theta \left[\frac{\mu}{2}\Phi_0^2 + \sqrt{\mu G}\Phi_0\Phi_+\Phi_- \right] + h.c. \right\} , \quad (2) \end{aligned}$$

where we have again put in a soft SUSY breaking term. Without the latter, the last line in the equation is an exact supersymmetrization of the NJL counterpart. Note that the superpotential reduces to

$$W = -\frac{\mu}{2}\Phi_0 \left[\Phi_0 + 2\sqrt{G/\mu}\Phi_+\Phi_- \right] , \quad (3)$$

to be compared against $W = -\mu\Phi_2(\Phi_1 + g\Phi_+\Phi_-)$ for the old SNJL model. The equation of motion for Φ_0 yields $\Phi_0 = -\sqrt{G/\mu}\Phi_+\Phi_-$, *i.e.* Φ_0 as a composite of two chiral superfields. Simultaneously, Φ_0 also takes the role of the effective Higgs superfield, supersymmetrizing the composite scalar field of the original NJL model. The mathematical structure of our Lagrangian thus resembles the latter more closely compared to that of the old SNJL model. In the latter case, Φ_1 is the composite while Φ_2 plays the Higgs.

To look at the physics of the model and its possible symmetry breaking feature, we follow a simple approach discussed in Ref. [7] for a gauged version of the old SNJL model. The approach is also discussed in Ref. [5] for the NJL model, where it is explicitly shown to give the same result as the gap equation analysis. For this purpose, we

calculate the effective two point function for the auxiliary superfield Φ_0 of the Lagrangian in Eq. (2). From the one-loop supergraph diagram with Yukawa vertices, we have, in the presence the extra soft SUSY breaking m^2 terms,

$$\begin{aligned} \Gamma_{eff} &\simeq \int d^4x d^4\theta \left(\frac{y^2}{16\pi^2} \log \left[\frac{\Lambda^2}{\mu_Q^2} \right] \right) \bar{\Phi}_0\Phi_0 [1 + 2m^2\theta^2\bar{\theta}^2] \\ &\equiv \int d^4x d^4\theta Z_0 \bar{\Phi}_0\Phi_0 [1 + 2m^2\theta^2\bar{\theta}^2] . \quad (4) \end{aligned}$$

Note that SUSY breaking mass induced for the effective canonical Higgs superfield $\sqrt{Z_0}\Phi_0$ is tachyonic, suggesting the possibility of radiatively induced symmetry breaking. Actually, the one Higgs case is of very limited interest. To have the Φ_0^2 term in the Lagrangian, Φ_0 has to be in a real representation of the model symmetry. The electroweak doublet needed for the SM symmetry breaking, for instance, cannot be modeled directly.

III. TOWARDS THE MSSM

The supersymmetric SM requires two Higgs superfields, instead of one. Consider a Lagrangian of four chiral superfields (actual the three third-generation quark superfields) with soft SUSY breaking masses and the following superpotential

$$W = G \varepsilon_{\alpha\beta} Q_3^{\alpha a} U_3^c{}^a Q_3^{\beta b} D_3^b (1 + A\theta^2) , \quad (5)$$

where we use standard notation for quark doublet and singlet superfields; α, β are $SU(2)$ indices and a, b are color indices. Two Higgs superfields are introduced to rewrite the superpotential, with the $SU(2)$ and color indices suppressed, as the equivalent

$$\begin{aligned} W &= \mu(H_d - \lambda_t Q_3 U_3^c)(H_u - \lambda_b Q_3 D_3^c)(1 + A\theta^2) \\ &= (-\mu H_d H_u + y_t Q_3 H_u U_3^c + y_b H_d Q_3 D_3^c)(1 + A\theta^2) , \quad (6) \end{aligned}$$

where $\mu\lambda_t = y_t$, $\mu\lambda_b = y_b$, $\mu\lambda_t\lambda_b = G$. The equation of motion for H_u gives $H_d = \lambda_t Q_3 U_3^c$ while that for H_d yields $H_u = \lambda_b Q_3 D_3^c$. Note that the SUSY breaking part with parameter A also gives the B term. Promoting the quark-superfield kinetic terms to full gauge kinetic terms and adding the pure gauge superfield terms, one arrives at a Lagrangian for third generation quark masses similar to the one considered in Ref. [8] for the old SNJL model. In our holomorphic model, however, both H_u and H_d come as quark-superfield composites, with gauge kinetic terms expected to be generated at low energy through the Yukawa couplings [7].

A full superpotential for the MSSM Lagrangian with the two effective Higgs superfields can be given by

$$W = G_{ijkh} Q_i U_j^c Q_k D_h^c (1 + A\theta^2) + G_{ij}^e Q_3 U_3^c L_i E_j^c (1 + A\theta^2) , \quad (7)$$

with the usual family indices. We assume only the coupling G_{3333} is strong enough to drive the dynamical symmetry breaking as described above. The A parameter does not have to be universal, but we do have the

MSSM A_t and A_b and the usual B -parameter originate from a single soft SUSY breaking parameter for the G_{3333} term, as in Eq. (5) above. The four-superfield terms with couplings G_{ij33} , G_{33kh} and G_{ij}^e give rise to the rest of the up-type quark, down-type quark, and charged lepton Yukawa interactions, respectively. The remaining G_{ijkh} terms are not needed. Neither does it hurt to have the extra G_{ij}^e type terms so as to include all terms admissible by the gauge symmetry.

At this point, it is interesting to go back and look at how the old SNJL model get to the MSSM itself. While some careful numerical studies have been performed (see, for example, Ref. [8]), apparently, the full Lagrangian has not been explicitly given at the level before introducing the auxiliary superfields. The original dimension-six term gives the composite $H_d \sim Q_3 U_3^c$ with the other auxiliary H_u to give up-type quark Yukawa terms. Duplicating the structure to generate the down-sector quark Yukawa terms from $\bar{Q} \bar{D}^c Q D^c$ operators (and charged lepton part from $\bar{L} \bar{E}^c L E^c$ terms) will introduce a further pair(s) of Higgs superfields. On the other hand, sticking to using $H_d \sim Q_3 U_3^c$ as the Higgs superfield for the down-sector quark and charged lepton Yukawa terms implies exactly that they come from the kind of holomorphic dimension-five operators introduced here. For example, $\int d^2\theta Y^e H_d L E^c \leftarrow \int d^2\theta (-g Y^e Q_3 U_3^c L E^c)$.

IV. RENORMALIZATION GROUP ANALYSIS

To see if the holomorphic SNJL model scenario discussed above is compatible with the low energy phenomenology and known experimental constraints, we perform a renormalization group (RG) analysis and report the first results here, leaving most of the details to a forthcoming paper [10]. Here, we focus only on the most important parameters, the top and bottom quark Yukawa couplings. We also take a look at the Higgs mass predicted.

The IQFP scenario had been the focus in most of the

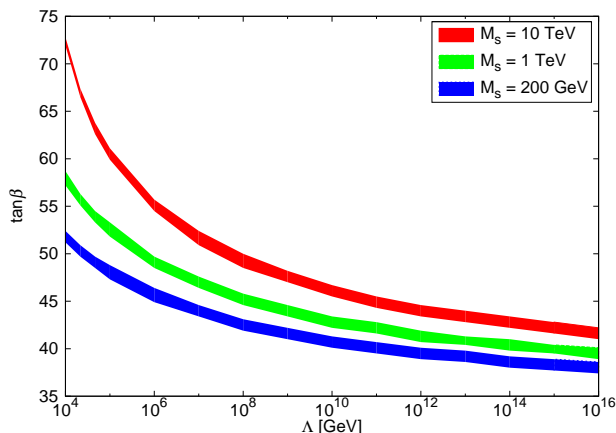


FIG. 1: Solution window for $\tan\beta$ vs Λ_b

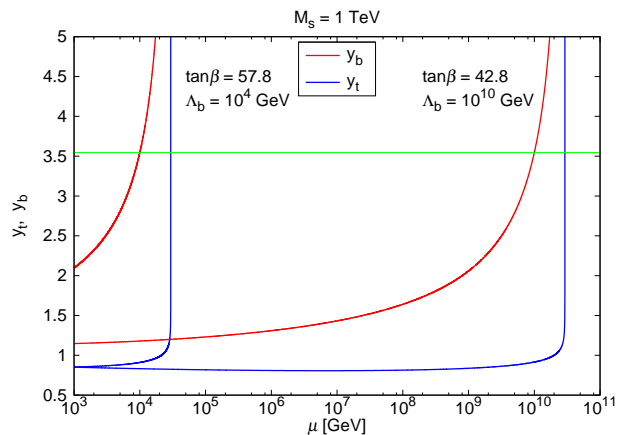


FIG. 2: Illustration of y_b and y_t runnings for a couple of cases.

RG analysis related to the NJL models in the literature. We first note that while the scenario has its appeal, especially when one aims at getting a prediction for the top mass, it is not strictly required by the NJL model mechanism. The latter requires only for the related Yukawa couplings, those for both the top and the bottom quark in our case, to blow up approaching the background scale Λ . As our analysis is based on one-loop RGEs, we may not reliably trace the running beyond the scale where the couplings pass the perturbative limit [4]. Denote the scales by Λ_t and Λ_b (*i.e.* where $y^2/4\pi = 1$), respectively. We look for admissible cases of Λ_t and Λ_b values with the now precisely determined top and bottom masses $[m_b = 4.20^{+0.17}_{-0.07} \text{ GeV}]$ implemented. Note that for large value of $\tan\beta$, the bottom Yukawa y_b is big. We find that there is always a window of $\tan\beta$ value giving admissible solution, for any Λ_b we take (with SUSY scale M_S from 200 GeV to 10 TeV), as given in Fig. 1. In Fig. 2, we show the y_t and y_b runnings for a couple of typical cases. Note that y_b plays the most important role, reaching the perturbative limit before y_t . We have $\Lambda_t \sim 3\Lambda_b$, with the one-loop RG showing divergent behavior for both y_t and y_b almost right beyond Λ_t . It is also important to note that we have included a SUSY threshold correction ϵ_b [*cf.* $(1 + \epsilon_b \tan\beta) = \sqrt{2}m_b/(y_b v \cos\beta)$] of value -0.01 in the running of y_b . The negative value is important. For $\epsilon_b > 0$, solution is possible only with uncomfortably large $\tan\beta$. The exact value of ϵ_b depends on the SUSY spectrum. For the simple case of a degenerate spectrum, we have $|\epsilon_b| = \alpha_s/3\pi \sim 0.01$ [11]. For the first result here, we take the ballpark value. Looking at the RG running only, the yet to be determined value of $\tan\beta$ means that it can be chosen to fix the low energy y_b input value to yield almost any Λ_b value. The negative ϵ_b help to reduce somewhat the $\tan\beta$ value thus taken, keeping y_t not too small to fit in the RG picture. That explains the main feature behind our result. If one takes a careful inspection of the admissible solution plot, one will see that for a large enough Λ_b , the $\tan\beta$ solution window loses sensitivity to further increase in Λ_b . This is what is

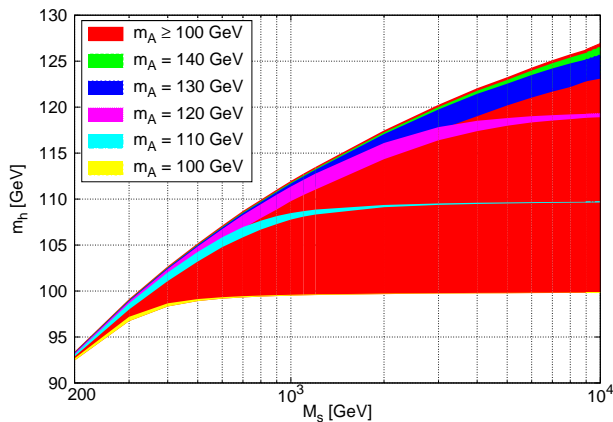


FIG. 3: Prediction for the lightest Higgs mass.

corresponding to the IQFP solution, as a variation of Λ_b translates into a variation of y_b at a fixed Λ .

For the determination of the Higgs mass, we follow the approach of Ref.[8]. The MSSM Higgs potential parameters for the quartic terms are to be fixed at the M_S scale by the gauge couplings, and then run towards the lower energy scale with the appropriate RG equations. Together with the $\tan\beta$ value, the Higgs potential, assumed to give the right electroweak symmetry breaking, is left with only one free parameter here taken as M_A , the pseudoscalar mass. We determine the lightest Higgs mass as a function of M_S for $M_A > 100$ GeV. The value loses sensitivity to M_A as the latter get bigger. In fact, for large M_A , a SM Higgs potential should rather be used with the only parameter fixed at M_S . We confirmed a good agreement of the result for the case. We present in Fig. 3 the result for the lightest Higgs mass. For $M_S < 1.5$ TeV, it is on the low side compared with the 114 GeV SM Higgs search limit, but with admissible values as MSSM Higgs [12]. The result has little sensitivity to Λ_b . We conclude that the Higgs mass we have is generally admissible, while further studies explicitly tuned to various parameter space region are needed, better to be done in conjunction with analyses of the full SUSY spectrum. It is plausible that the Higgs mass will be increased after further corrections are taken into account.

V. FINAL REMARKS

We have explored above the idea of having a holomorphic SNJL model. The structure is unconventional and provocative in the sense that it actually requires a bi-

scalar vacuum condensate to give symmetry breaking and Dirac masses. The bi-fermion composite/condensate setting was inspired by the Cooper pair and the BCS theory of superconductivity, and quite well explored with various approaches [5]. Our discussion for the bi-scalar condensate scenario in this paper is short of establishing it on a similar ground. The scenario, however, may provide a new direction for modeling dynamical symmetry breaking in a supersymmetric setting.

We have discussed a holomorphic SNJL model with two Higgs superfields giving complete MSSM Lagrangian as the low energy effective theory. The latter may realize the IQFP solution for third family fermion masses as studied in Ref. [9], even for $m_t = 171.3 \pm 1.6$ GeV. More interestingly, a much lower background scale Λ for the SNJL model, while deviating from the fixed point picture, still gives admissible solution for a somewhat Λ -sensitive window of $\tan\beta$. Even Λ of the TeV order may be admissible. For $M_S < 1.5$ TeV, the Higgs mass is on the low side, but compatible with the MSSM Higgs search limits. The model also prefers ϵ_b to be negative.

The RG analysis presented here includes only the first results. There are other interesting parts to be investigated, for example, the behavior and preferred values of the soft SUSY breaking parameters. Detailed comparison with the SUSY-top condensate scenarios should also be studied. More results will be given in a forthcoming publication [10].

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Note Added:- After we posted the first version of this paper, we were kindly informed by Kobayashi and Terao about their study on a supersymmetric QCD model with a complicated Higgs sector[13], with Higgs superfields essentially in a real representation. Seeking to eliminate the Higgs superfields as auxiliary composites, the author arrived at the dimension-five operator to be taken as the source of the Higgs superfields and the origin of the (dynamical) symmetry breaking. This is essentially the holomorphic SNJL model, which we rediscovered and analyzed from the basic supersymmetrizing perspective. It is easy to see from our discussion above that assuming essentially only one Higgs superfield multiplet cannot fit into the conventional SNJL model with the dimension-six operator, but works with the holomorphic model.

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